between a measured primary or secondary antenna pattern and a calculated one.

7) Rapid insight is obtained with regard to the power distribution of $D(\theta, \phi)$ per unit solid angle.

8) Spillover may be computed directly without the use of time-consuming methods, such as planimeters or special plotting papers [12].

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Frequency-Domain Characterization of Microwave Delay Lines

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***Abstract*—Techniques for evaluating delay lines with an automatic network analyzer are described. Extremely precise values of the delay can be obtained by an iterative process which uses final measurement frequencies spaced by integral multiples of the reciprocal delay.**

The magnitude of leakage and triple-delayed transmissions are determined from measurements which are also necessary to obtain accurate values of insertion loss.

Introduction

UPON INITIAL consideration, one might suppose that delay lines should be characterized in the time domain. However, computer-controlled frequency-domain measuring equipment can evaluate delay lines rapidly. The computer permits an efficient set of test frequencies to be determined from preliminary measurements and transforms the results back into time-domain characteristics.

The automatic equipment, exemplified by the Hewlett-Packard 8542A system, sets a signal generator to a discrete, digitally prescribed frequency for each measurement. (The microwave path of this system, located at Computer Metrics, Inc., Rochelle Park, N. J., was revised to extend the dynamic range by 20 dB.) In most applications of this equipment, measurements are taken at equally spaced frequencies over the range of interest. In delay-line measurements such frequency choices will often result in poor estimates of delay and attenuation. Undelayed leakage and triple-delayed signals (arising from multiple reflection within the delay medium) give the transmission function a fine structure in the frequency domain. The errors are analogous to the "aliasing" errors incurred in sampling a time function too infrequently in relation to the high-frequency content of the time function.

The equipment mentioned requires about 250 ms to set the signal generator at each new frequency. It is generally impractical to measure throughout a frequency band at frequencies spaced closely enough to avoid the aliasing errors. However, appropriately chosen close-spaced groups of frequencies permit determination of the true delay and attenuation, and also

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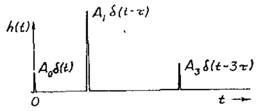

Fig. 1. Idealized response of delay line to impulse.

the magnitudes of the leakage and triple-delayed signals.

Frequency-domain characterization is particularly appropriate for delay lines with bandpass characteristics. Even when the delay line is a fairly broad-band device, it may be used with band-limited signals. The frequency-domain measurement in the band-of-use gives precise, relevant information without confusion from out-of-band characteristics. Frequency-domain measurements are reproducible even for dispersive delay lines.

Previously, RF pulses have been used for routine measurements of these quantities for band-limited applications. The pulse method is not as suitable as the CW technique for a detailed examination of the dispersion of the delay line, because the pulsed signal does not have a well-defined frequency. Readout for the pulsed method is usually visual observation on an oscilloscope, a circumstance which minimizes the chances of gross errors, but is not conducive to high accuracy.

At first glance it would seem obvious that the separation, in time, of the faint leakage and triple-delayed signals from the relatively strong delayed signal would be a clear-cut advantage of the pulsed RF method. However, the great sensitivity of CW techniques provides adequate dynamic range for the distinctive time characteristics of the faint signals to be transformed into equally distinctive features of the frequency spectrum.

Single-ended delay lines have been characterized by impedance measurements, and the importance of the "fine structure" in the frequency response has been pointed out [1].

Model and Analysis

The idealized time-domain response of the delay line to an impulse-function input is shown in Fig. 1. Of course this figure does not represent the actual response of the delay line to an impulse. It is, instead, the idealized standard by which a group of measurements in a narrow frequency range are to be judged. That is our definitions of leakage, delay, and triple delay at a particular frequency are based on this idealized standard.

The mathematical representation of the idealized transfer function of the delay line is

$$h(t) = A_0\delta(t) + A_1\delta(t-\tau) + A_3\delta(t-3\tau) \tag{1}$$

where A_0 is the leakage. A_1 is the desired delayed signal, and A_3 is the triple-delayed signal. By straightforward Fourier transformation, we find the transfer function in the frequency domain:

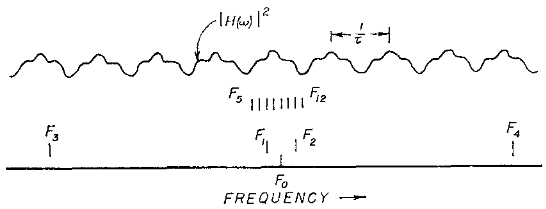

Fig. 2. Transmission coefficient (magnitude)² versus frequency. F_0 equals nominal frequency. F_1, F_2 (with F_0) used to determine approximate delay. F_3 and F_4 give vernier delay. F_5 to F_{12} are used to determine loss, leakage, and triple-delay magnitudes.

$$\begin{aligned} H(\omega) &= A_0 + A_1e^{-j\omega\tau} + A_3e^{-j3\omega\tau} \\ &= A_1e^{-j\omega\tau}\left[1 + \frac{A_0}{A_1}e^{j\omega\tau} + \frac{A_3}{A_1}e^{-j2\omega\tau}\right] \end{aligned} \tag{2}$$

and the power transmission is

$$\begin{aligned} |H(\omega)|^2 = A_1^2\Bigg[1 &+ 2\frac{A_0}{A_1}\cos\omega\tau + 2\frac{A_3}{A_1}\cos 2\omega\tau \\ &+ 2\frac{A_0A_3}{A_1^2}\cos 3\omega\tau + \left(\frac{A_0}{A_1}\right)^2 + \left(\frac{A_3}{A_1}\right)^2\Bigg]. \end{aligned} \tag{3}$$

For practical delay lines,

$$\left(\frac{A_0}{A_1}\right)^2 + \left(\frac{A_3}{A_1}\right)^2 \approx 0. \tag{4}$$

One can see from (3) that the transmission magnitude has a fine structure, as indicated in Fig. 2. The "period" of this fine structure in the frequency domain is equal to the reciprocal of the delay time (assuming that the leakage A_0 is not zero). The pattern can be expected to have a second harmonic content resulting from triple delay.

For example, a microwave delay line with a 2.5-μs delay will have a frequency-domain transmission coefficient that oscillates with a "period" of 0.400 MHz. If such a delay line is to be used between 2 and 4 GHz, one would be tempted to evaluate it with frequencies spaced by considerably more than 0.4 MHz. But the results for loss and delay would then be dependent in a rather random way on the relation between the arbitrarily chosen frequency step size and the exact value of the delay time. In a measured example, the peak-to-peak variation in apparent loss was 8 dB out of 40 dB.

The delay τ itself can be conveniently obtained by a special choice of measurement frequencies

$$\tau = -\frac{\Delta\phi}{2\pi\Delta f}, \qquad \text{for } \Delta f = \frac{\text{integer}}{\tau} \tag{5}$$

where $\Delta\phi$ is the change in phase for two measurements separated by Δf in frequency. One can see that (5) is a prescription for the following iterative process.

Description of the Method

First it is worthwhile to give the automatic system the benefit of our initial knowledge by inputting a *maximum* expected delay. Thus if 3-μs delay lines are being measured, one can reasonably specify a maximum delay of 5 μs.

Since the frequency-domain measuring system is unable simply to resolve ambiguities due to phase changes greater than 360°, the computer first determines a frequency difference that ensures that the phase change will be less than 360° for the maximum expected delay. The apparatus is then at first constrained to measure phase differences at frequency intervals no larger than this step. Two such steps are combined to get the first approximate measured value of delay time. This value permits improving the accuracy of the delay measurement in two ways.

1) It is now possible to use a phase change of many times 360° (e.g., eight times), since the number of full revolutions of the transmission phasor can be inferred from the approximate value of the delay. A vernier type of accuracy enhancement is thus obtained.

2) The frequencies can be chosen to be corresponding points in the cyclic frequency variation of the transmission coefficient, as required for (5).

This process of improving accuracy in the delay measurement can be repeated until Δf approaches the desired frequency resolution. However, one iteration generally suffices to give four significant decimal digits. It is questionable whether greater accuracy is meaningful for practical devices afflicted with leakage, triple delay, and dispersion.

From the accurate value of the delay, it is possible to formulate convenient frequency groups to evaluate the coefficients of (3). Eight equally spaced samples are used in the final program, as shown in Fig. 2.

We have found that Fourier analysis of the power transmission, (3), gives a good evaluation of the parameters A_0/A_1 and A_3/A_1, and a mean value representing the true loss. Time-domain measurements on nondispersive samples were used in the comparison. The alternate approach of using (2) is possible, since phase data are obtained in the measurement. An accuracy comparison of the two approaches has not been made.

The program was written in Basic and requires about 10 s at each nominal frequency for measurement, calculation, and printout.

Results

In the S-band region, a 3-μs line was evaluated which had losses of 35 dB. Leakage was 30 dB below the delayed pulse. Triple-delayed output was 35 dB below the delayed pulse. The results were unaffected by a 20-dB improvement in dynamic range through system modification.

Delay lines from 1 to 64 μs were evaluated in the VHF range 45 to 200 MHz.[1] For example, an 8-μs line with 50 dB of loss was found to have leakage 50 to 60 dB below the main signal and triple delay 45 to 55 dB below the main signal (i.e., 95 to 105 dB below the input). For reproducible results, a 20-dB improvement in dynamic range was barely sufficient. A 30-dB transistor preamplifier on the receiving side permitted stable and reproducible leakage and triple-delay results to be obtained.

The results agree with pulsed RF determination within the confidence limits of the latter method. Ultimate accuracy is dependent upon frequency accuracy, which will be one part in 10^7 under almost all circumstances and can be raised to one part in 10^8 with care and independent testing. At one part in 10^7, a 0.2-percent error would be expected in a 3-μs delay measured at 36 Hz with a frequency difference of $8/\tau$. Note, however, that for accurate work one should not compute the delay from the frequency commands, but from the actual digitally set frequencies that result from the commands.

[1] The normal range of the system is 100 to 18 000 MHz, but a modification permits operation at lower frequencies.

Conclusions

Consistent values of the loss of a delay line can be obtained from frequency-domain measurements, if enough measurements are made to determine the leakage and triple-delayed components. The measurements and calculations can be carried out with available automatic equipment, with minor modification, at practical production rates. With modest additional attention to frequency accuracy, the method could well be the standard by which other methods would be judged.

A natural definition for "delay time" at a particular frequency is the rate of decrease of phase with frequency, i.e., "group delay." The problems and opportunities involved in measuring this quantity for delay lines have been described.

In analogous fashion, definitions for leakage and triple-delayed components at a particular frequency have been proposed and used as a basis for measurement.

While measurement-system leakage can contribute to the undelayed leakage, there is no time delay in the system comparable to the 2τ difference between the delayed and triple-delayed signals. Therefore, the triple-delayed magnitude results are not much affected by system leakage.

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